Appendix

Part C: Dedekind-MacNeille Completion

In this section we want to try understand what exactly happens to a poset in a Dedekind-MacNeille Completion. Our theoretical bases is Proposition A.13 and those definitions belonging to it.

To see how we can "conjure up" a lattice from any arbitrary poset it is best we look at a concrete example. Our starting position is the **P** = (X, P) in Figure C.1.

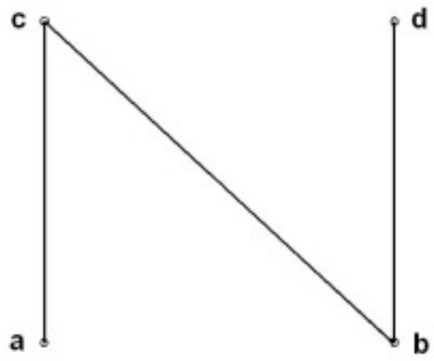

Figure C.1: A poset **P**

P is definitely not a lattice, because, for example, elements c and d do not have a supremum. So, it makes sense to ask for a completion which then has to be a (complete) lattice.

We are interested in the image of basic set X under the operator Γ_{UL}. Thus, we have to calculate the values of Γ_{UL} for all subsets of X:

Subset A of X	A^U	$\Gamma_{UL}(A) = (A^U)^L$
{a}	{a, c}	{a}
{b}	{b, c, d}	{b}
{c}	{c}	{a, b, c}
{d}	{d}	{b, d}
{a, b}	{c}	{a, b, c}
{a, c}	{c}	{a, b, c}
{a, d}	∅	{a, b, c, d}
{b, c}	{c}	{a, b, c}

$\{b, d\}$	$\{d\}$	$\{b, d\}$
$\{c, d\}$	$\varnothing$	$\{a, b, c, d\}$
$\{a, b, c\}$	$\{c\}$	$\{a, b, c\}$
$\{a, b, d\}$	$\varnothing$	$\{a, b, c, d\}$
$\{a, c, d\}$	$\varnothing$	$\{a, b, c, d\}$
$\{b, c, d\}$	$\varnothing$	$\{a, b, c, d\}$
$\{a, b, c, d\}$	$\varnothing$	$\{a, b, c, d\}$

The operator Γ_{UL} produces the set

$$im(\Gamma_{UL}) = \{\varnothing, \{a\}, \{b\}, \{b, d\}, \{a, b, c\}, \{a, b, c, d\}\}.$$

Proposition A.13 - Dedekind-MacNeille tells us that all that's left to do to obtain our (complete) lattice which densly embeds our poset **P**, iis to sort the elements by set inclusion $\subseteq$. Figure C.2 below illustrates our result.

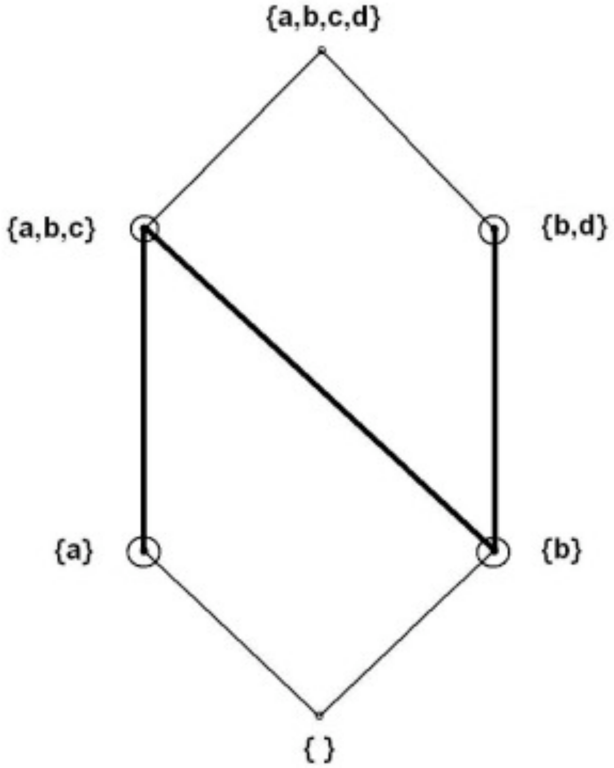

Figure C.2: Dedekind-MacNeille completion of the poset **P**

Clearly illustrated by the figure is the embedding through function ϕ_X which assigns to each element in X its Down-set ($\phi_X(a) = \{a\}$, $\phi_X(b) = \{b\}$, $\phi_X(c) = \{a, b, c\}$, $\phi_X(d) = \{b, d\}$).

The computer program "Algebra Workbench" (AWB) was created by Markus Sprenger. The documentation found here is based on a 2005 master thesis by Christoph Röthlisberger. The translation and adaptation of the material was done by Cindy-Jane Armbruster.

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