Definitions and Propositions

Part A: Order Theory 4

Before wrapping up the section on order theory and before moving on to definitions and propositions pertaining to universal algebras, we want to show you a nice application of critical and weakly critical pairs.

Definition A.17 - Critical Pair

P = (X, P) is a poset. (x,y) is a *critical pair* if the following conditions hold:

1. x and y are incomparable
2. $a < x \Rightarrow a < y \; \forall a \in X$
3. $b > y \Rightarrow b > x \; \forall b \in X$

Remark A.18

(x,y) critical pair $\not\Rightarrow$ (y,x) critical pair.

Definition A.19 - Weakly Critical Pair

P = (X, P) is a poset. (x,y) is a *weakly critical pair* (also called *subcritical pair*) if the following conditions hold:

1. $x \nleq y$
2. $a < x \Rightarrow a \leq y \; \forall a \in X$
3. $b > y \Rightarrow b \geq x \; \forall b \in X$

Remark A.20

Every critical pair is also a weakly critical pair. In the following we call a weakly critical pair which is not critical *proper* weakly critical.

Definition A.21 - Maximal 0-1 Sublattice

L is a finite, distributive lattice (***L*** $\in DL_{fin}$) and $M \subseteq L$ is a subset.

M is a *0-1 sublattice* of ***L*** if **M** is a sublattice of ***L*** with $0_L, 1_L \in$ **M**.

M is a *maximal 0-1 sublattice* of ***L*** if **M** is a 0-1 sublattice and there is one 0-1 sublattice ***M'*** for which the following holds: **M** ⊆ **M'** ⊆ ***L*** ⇒ **M'**=**M** or **M'**=***L***.

Definition A.22 - ∨-, ∧-irreducible Element, *J(L)*

L is a finite distributive lattice. $x \in L$ is ∨*-irreducible* (join-irreducible) if and only if $x = y \vee z \Rightarrow x = y$ or $x = z$ (definition of $\wedge$-*irreducible* (meet-irreducible) is analogous).

J(L) is the set of all $\vee$-irreducible elements of ***L*** without 0.

The dual set ***M(L)*** (Meet-irreducible Lattices) is defined as the set of all $\wedge$-irreduzible elements of ***L*** without 0.

Due to the symmetric nature of lattices a definition of only ***J(L)*** is sufficient.

Proposition A.23 - Maximal Sublattices

Maximal 0-1 sublattices of ***L*** correspond bijektively to the critical and weakly critical pairs in ***J(L)***.

The computer program "Algebra Workbench" (AWB) was created by Markus Sprenger. The documentation found here is based on a 2005 master thesis by Christoph Röthlisberger. The translation and adaptation of the material was done by Cindy-Jane Armbruster.

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